Review

Researches on fluid dynamics of centrifugal compressors

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Abstract: Turbo-machines (e.g. compressors, blowers and pumps) obtain energy transfer from their rotating impellers to fluid flowing through them, and they are used for a wide range of purposes obtaining pressurized fluid flow. The author created theoretical models to clearly describe the complicated flow phenomena in impellers and in diffusers, based on the fluid dynamic theory. Furthermore, he succeeded in inventing a new diffuser to control a part of the complicate flow, which improved performance of compressors dramatically.

Key words: Turbo-machine; centrifugal-compressor; impeller; diffuser; tip-clearance; boundary layer.

Introduction. Centrifugal compressors are widely used for distributing city gas, for transporting natural gas through long pipelines, for liquidizing natural gas, for pressurizing gas at various kinds of chemical plants. Also, they are indispensable as turbo-superchargers for reciprocating engines to increase output power dramatically. In many of these cases, demand for flow rate at high pressure varies during operation at a fixed shaft speed.

As complex configurations of turbo-machines make it difficult to analyze the behavior of fluid flow through them theoretically, a lot of empirical descriptions have been used for design. If better understanding on flow in turbo-machines makes it possible to design machines with better efficiency, a considerable amount of energy resources will be saved throughout the world.

Basic problems on turbo-machine. As opposed to reciprocating compressors, which are popular as high pressure-ratio and low capacity compressors, turbo-compressors are compact for the capacity and quiet without vibration, as they continuously pressurize fluid by means of high-speed rotating impeller. Consequently they are widely used and especially for large capacity applications in industry turbo-compressors are exclusively adopted.

In steady flow, fluid flows toward a zone of low pressure, being accelerated by favorable pressure gradient, but in a diverging passage flow increases the pressure by reducing the velocity. In cases of centrifugal compressors in Fig. 1, this principle is directly applicable only to the diffuser. Flow through a rotating impeller is not steady unless it is observe in the rotating system. That is, if relative velocity in an impeller is reduced, static pressure increases. Using this principle the pressure of flow through a compressor increases pressure both in the impeller and in the diffuser by reducing the relative velocity successively.

In cases of axial compressors where diameter of impeller and diffuser remains constant from the inlet to the exit, phenomena induced by rotating impeller and by stationary diffuser are identical to those in cases of centrifugal compressors except that the flow remains on a cylindrical surface.

Inlet geometry of vanes. Specifically, in the case of a rotating impeller, straight axial or radial flow toward a rotating impeller is recognized as diagonally incoming flow relative to the impeller. Therefore, at the inlet all the vanes of the impeller should be oriented toward the direction of incoming flow relative to the impeller. Furthermore, the vanes must be curved smoothly toward axial or radial direction so that passages between vanes become wider downstream. Consequently fluid increases pressure while it flows

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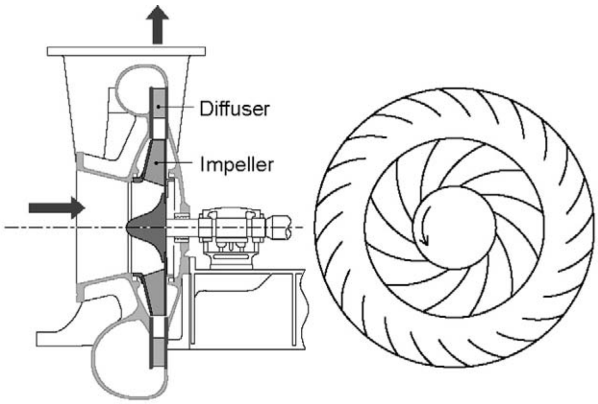

Fig. 1. Centrifugal compressor. (Impeller with two-dimensional vanes, Diffuser with vanes.)

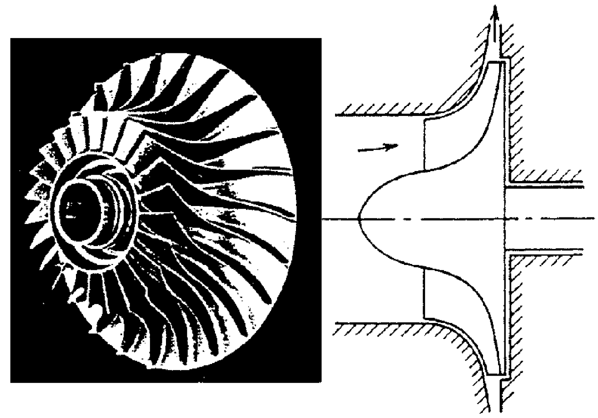

Fig. 2. Impeller with three-dimensional vanes.

through the impeller.

Fluid in the impeller rotates with the impeller, and it flows out of the impeller keeping strong swirl. Downstream of the impeller is a parallel wall diffuser without or with many fixed vanes. If there are vanes, leading edges of vanes are oriented to the direction of flow out of the impeller, and vanes are bended so that the flow reduces its swirl velocity component generating conversion from dynamic pressure to static pressure.

As flow through a compressor successively increases pressure both in the impeller and in the diffuser, a turbo-compressor is suitable for pressurizing a large amount of flow. Furthermore, if the diameter of the impeller at the exit is larger than that at the inlet, the centrifugal acceleration working on fluid in the rotating impeller serves to increase the exit pressure further, in addition to the increment of pressure based on reduction of the velocity passing through.

Load on vanes. By the way, the dynamic pressure of air at a velocity 40 m/s, a typical velocity of hurricane, is equal to the pressure under a 100 mm water column, which is one hundredth of the increment of pressure to double the atmospheric pressure. In order to achieve a large pressure ratio, a big change of velocity is required, i.e. long vanes and a high tip speed of an impeller are required to a turbo-compressor. If it is desired to make the diameter of an impeller as small as possible, the leading edge of vane is advanced toward the suction pipe as shown in Fig. 2. Then vanes of impeller are three-dimensional.

In order to make the configuration of centrifugal impellers simple and robust, two-dimensional vanes are preferable, where the inlet flow to the impeller zone is almost radial without swirl. In cases where a very large pressure ratio is required, several units of impeller and diffuser are mounted in series on an axis to add series of pressure-rise by these units. However, there are many other problems to overcome for a compressor to fulfill requirements for stable operation at required conditions.

Flow in boundary layer against adverse pressure gradient. In any flow through a passage, a layer of low velocity, so-called a boundary layer, grows along the wall due to the wall friction, where velocity varies from zero on the wall to the main flow velocity.

In flow through a diverging passage, the pressure gradient balances with a moderate change of velocity in the main flow, but the pressure gradient is too steep for the slow flow in the boundary layer to keep the profile of velocity distribution. As a result the profile of velocity in the boundary layer is distorted further and the layer gets thick absorbing a part of the main flow. In the worst case, reverse flow occurs near the wall, and the pressure hardly increases downstream. Rate of growth of the boundary layer and distortion of the velocity profile depend on the pressure gradient, and the rate of growth is also proportional to the initial thickness of the boundary layer.[1)-3)]

In cases of flow through curved ducts with rectangular cross section, there is pressure gradient perpendicular to the curved wall, and in the boundary layer on the flat side-walls cross-flow is induced toward the lower pressure zone and slow flow accumulates.[4),5)] Consequently, the boundary layer on the convex wall grows. Similar problems occur in turbo-machines. In order to limit growth of boundary layer in an impeller and in a diffuser as small as possible, care should be taken to the pressure distribution along the main flow

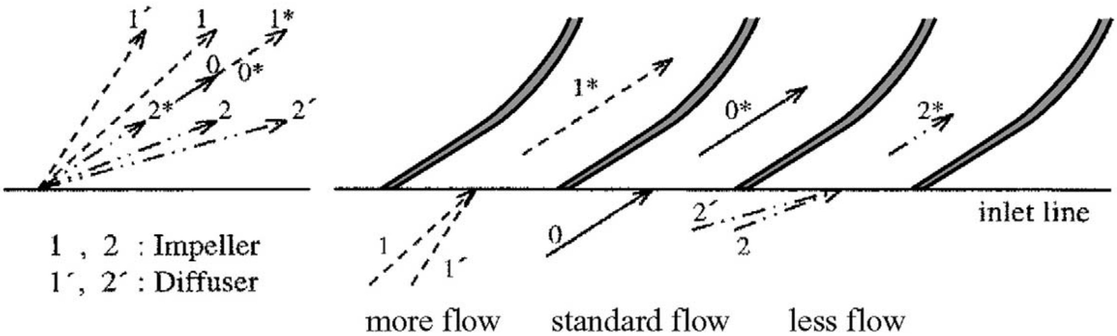

Fig. 3 Change of velocity vectors at the inlet of inclined vanes. (Two inlet conditions at three flow rates.)

and the distribution of boundary layer thickness on vanes, which is influenced by the cross flow in the boundary layer on the side-walls.

Entering flow to a row of inclined vanes. Regarding inlet part of impellers, the approaching flow toward the impeller without swirl, whether it is radial or axial, is recognized as a circumferentially inclined relative to the rotating impeller. Therefore, as shown in Fig. 1, inlet part of an impeller is made of a row of inclined vanes oriented to the incoming flow. Regarding inlet flow to a diffuser which is located downstream of an impeller, fluid in the impeller rotates with the impeller and it flows out into the diffuser as a swirl flow. In either case, at the entrance of a row of vanes, vanes are oriented to the direction of incoming flow at the standard condition. As shown in Fig. 3, the entry zone of these passages form a semi-open triangle, which consists of the surface of a vane and the throat, an open space between the vane and the leading edge of the adjacent vane, and a diverging passage follows the throat. The velocity vector 0 in the middle channel in Fig. 3 represents the inlet velocity vector at the standard condition.

Increased flow rate. If the flow rate increases at a fixed impeller speed, as shown in the left channel, direction of the incoming velocity-vector to the impeller vanes varies from vector 0 to vector 1 but magnitude of the velocity varies only a little. That is to say, it is almost equal to the magnitude of the standard inlet velocity vector 0 which is designed to be nearly equal to the velocity vector 0^* at the throat at the standard flow rate. On the other hand, the mean velocity at the throat 1^* and the mean velocity at the exit of the impeller vanes vary in proportion to the flow rate. Consequently, as the flow rate increases, the ratio of velocities at the exit versus at the inlet of the impeller becomes larger, and the increment of pressure through the impeller is reduced. That is, the pressure-rise through the impeller becomes less as the flow rate increases.

In the case of a diffuser, based on the general characteristic of impellers, when the flow rate increases, the swirl velocity vector 1' out of the impeller is reduced considerably, while the mean velocity vector 1^* at the throat and the mean velocity at the exit vary in proportion to the flow rate. Therefore, the reduction of pressure-rise across the row of vanes due to an increment of flow rate is more serious in the case of diffuser than that in the case of impeller.

Reduced flow rate. As the flow rate is reduced, the direction of velocity-vector 2 entering the impeller becomes more tangential than the direction of the vanes as shown in the right channel, and it cannot be deflected unless low pressure zone pulls the incoming flow along the vane. That is, the pressure along the vane in the semi open space must be reduced. On the other hand, if the velocity across the throat is uniform, magnitude of the mean velocity-vector 2^* at the throat section would be reduced and the pressure would rise. Therefore, the velocity is not uniform across the throat, and if the distortion of the velocity distribution exceeds an allowable limit, the pressure recovery in the diverging passages between the impeller vanes is hardly expected.

In cases of diffuser similar problems occur to the incoming velocity vector 2' to the diffuser, and the problem is more serious than the case of impeller as shown in Fig. 3. Obviously, the critical flow rate for good pressure recovery is the lower limit for the operating range of a compressor.

Velocity distribution in an impeller. There are two kinds of impellers, one with two-dimensional vanes and the other with three-dimensional vanes. Two-dimensional vanes are fixed between a hub disc and a shroud ring in an impeller as shown in Fig. 1, which is robust and easy to make. If the leading edges of vanes are advanced toward the suction pipe as shown in Fig. 2, vanes of the impeller are three-dimensional and the flow through it is very complicated, while incoming flow relative to the vanes must smoothly change the direction toward the radial part of the impeller vanes adding certain strength of swirl.

As three-dimensional vanes are complicated in shape and hard to make, usually only one side edge of vanes is built in the hub disc and the other side edge is left open. Regarding the open end, side edges of these vanes face closely the inner wall of the casing keeping a very small clearance in between to minimize leakage flow.

The vanes of impeller force flow to turn to the direction of rotation. Therefore, the pressure on the front side of each vane is higher than the pressure on the back. The load on vanes, i.e. the pressure difference between front and back of a vane, varies along the length of vane and it disappears at the exit of the impeller.

Load on vanes. As the load on vanes decreases near the exit, the direction of flow at the exit deviates backward relative to the direction of the vane, and the angle of deviation varies depending upon the shape and the number of vanes as well as the flow rate. The relationship between these parameters has been analyzed[6] for impellers with vanes of two-dimensional simple geometry assuming non-viscous fluid. However, in practice for design, evaluation is mostly based upon experimental data in the literature.[7]

Analysis on flow in impeller. For non-viscous fluid, flow in rotating impellers with vanes of three-dimensional geometry has been analyzed by means of a combination of two kinds of analysis. One is a theory to evaluate the distribution of velocity between vanes, or the distribution of load along vanes on an arbitrary surface of revolution,[8] and the other is an analysis on distribution of stream surfaces of revolution between given hub and shroud geometries, where the distribution of load along the length of each stream surface of revolution is given. Then a computer program was developed to combine these two analyses by means of iteration.[9]

The program has been widely used in industry for designing centrifugal compressors and pumps. Recently computer programs to analyze viscous flow through an impeller with any geometry are available. However, the computer program on non-viscous flow is applied for rational design of several candidate impellers, as it can be done quickly, then the viscous flow analysis is applied to finalize the design of the impeller.

Cross flow in boundary layer and accumulation. In order to swirl a flow by a rotating impeller, the pressure in front of vanes is higher than that in back, that is, there is pressure gradient between vanes. Therefore, in the boundary layer along the disc and the shroud ring of the impeller, the pressure gradient between vanes induces a cross flow toward the back surface of vanes and low velocity fluid accumulates. As a result the boundary layer on the back surface of vane gets thick.[4),5),10)] Furthermore, the pressure gradient toward the exit of impeller is very steep along the back surface of vanes. Therefore, the boundary layer on the back surface grows quickly toward the exit and a low velocity zone sometimes occupies considerable part of a pitch of vanes at the exit of the impeller.[11]

Clearance at side-edge of impeller-vanes. In cases of impellers without shroud ring to fix side-edge of impeller vanes as shown in Fig. 2 and Fig. 4, there is a narrow clearance between side-edge of impeller-vanes and the wall of stationary casing. In cases of axial compressors in Fig. 5, it corresponds to the clearance at the outer edge of vanes which is called the tip clearance. "Tip" may have a meaning of "the outer diameter". However, in order to generalize the discussion to include axial compressors, the clearance at the side-edge of impeller-vanes has also been called the tip clearance in the literature.

For predicting the loss due to the tip clearance of turbo-machines, different formula have been published in the literature based on different principles as listed in reference.[12] In 1986 the present author postulated a new theory[13] in which the pressure loss consisted of two parts. Due to the pressure difference between front and back surfaces of an impeller-vane, there is leakage through the clearance, and the leak flow is forced to increase the velocity along the vane. It is a cause of pressure loss. In addition, between the stationary casing and an imaginary surface of revolution covering side-edges of impeller-vanes, there is a thin annular space. In that space, no vane exists to support the meridian pressure difference between the exit and the inlet of the impeller. In order to prevent flow from leaking through this annular space, a shear force is required. It reduces the exit pressure and a pressure loss occurs.

It was suggested by audience that the two losses were the same loss, which were described from different point of view, or at least a part of the former was included in the latter or vice versa. However, it is clear

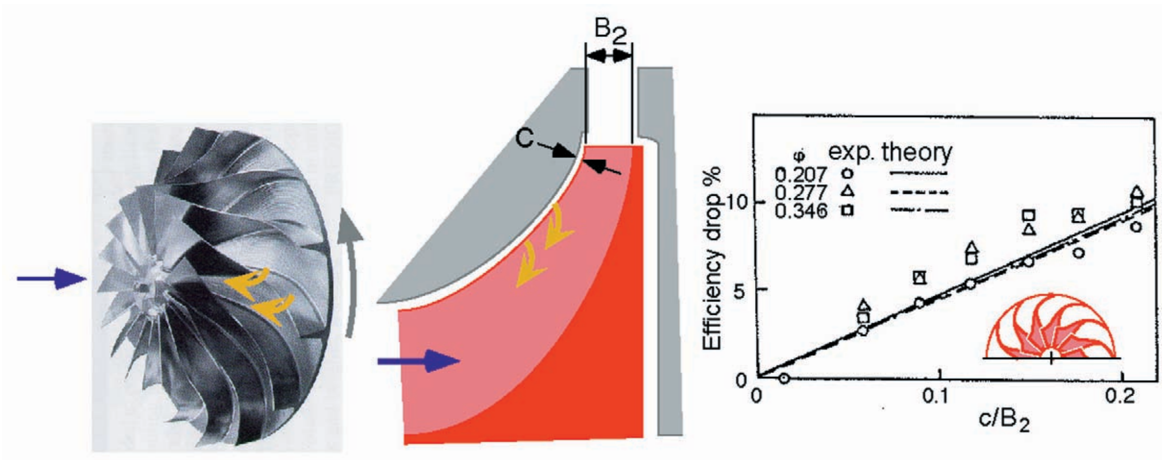

Fig. 4. Tip clearance effects on centrifugal impellers. (Three flow-rates.)

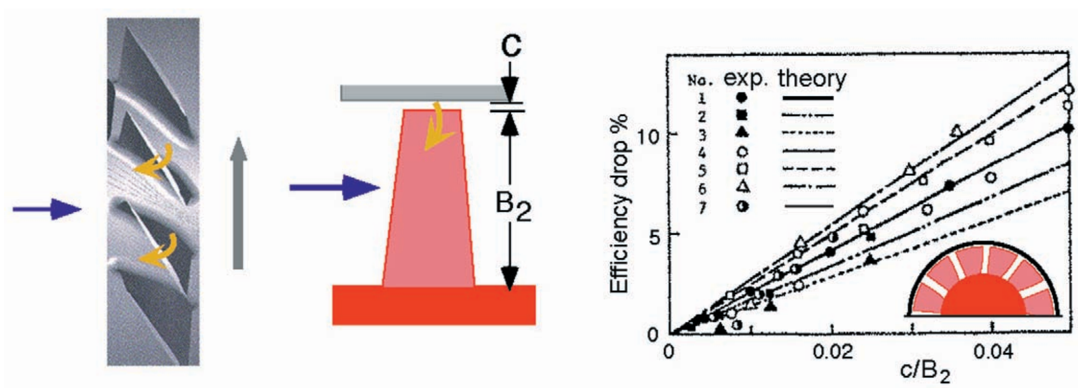

Fig. 5. Tip clearance effects on axial impellers. (Seven impellers.)

that the former comes from mixing of two different velocities parallel to the vane, while the latter comes from absence of the force normal to the vane in the annular clearance zone. These two forces are mutually perpendicular, therefore the two losses are entirely different in nature and they do not even partially overlap.[13] Furthermore, it is quantitatively made clear how the loss of kinetic energy of the leak flow is related to the drag induced by the leak flow through the clearance.

Experimental evidence. Regarding centrifugal compressors, performance change due to the tip clearance has been experimentally examined very thoroughly in the literature. Senoo *et al.* developed a theory to predict the effects of tip clearance at different flow rates and at different shaft speeds, and good agreement with experimental data were demonstrated as shown in Fig. 4.[13] In the literature, many experimental data on axial compressors are also available. Those data are compared with the theory and good agreement is demonstrated as shown in Fig. 5.[13] Furthermore, experimental data on three types of centrifugal compressors[14)-16)] designed for pressure ratio of about 6 were compared with the prediction and good agreement was observed.[17]

Characteristic tendency. Regarding efficiency drop due to the tip clearance, these examples demonstrate the following tendency with respect to various parameters. If the ratio of the clearance to the width at the impeller exit is equal, drop of efficiency due to the clearance is less for compressors with high-pressure-ratio than that for compressors with low-pressure-ratio. The magnitude of efficiency drop due to clearance becomes smaller as the flow rate is reduced and also at a

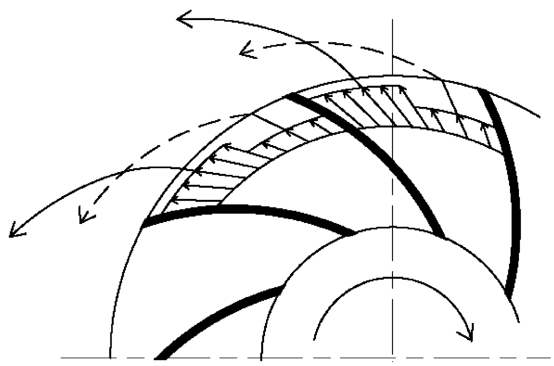

Fig. 6. Behavior of distorted flow downstream of a rotating impeller.

reduced shaft speed. These tendencies are clearly shown in equations presented in the theory.[17)]

Since load on vanes varies with the condition of operation, good agreement between experimental data and the prediction in a wide range of operation for several impellers means that the relationship between the losses and the load on vanes used in the prediction well represents the mechanics of pressure loss due to the clearance at the tip of vanes.[18),19)]

Behavior of distorted flow at the impeller exit. As mentioned before, on both side walls of an impeller, the boundary layer fluid between vanes moves forward to the back surface of the front vane and accumulates.[10)] Consequently in each passage between vanes a low velocity zone and a high velocity zone are formed as shown in Fig. 6. As they rotate with the impeller, a cyclic flow fluctuation is observed in the static system. However, according to experimental data, it is reported that the fluctuation is quickly attenuated in the annular space of the vane-less diffuser to become an axsymmetric flow, being accompanied with an increase of static pressure and an inherent loss of the total pressure.[11)]

Physics on unification. In any case, the periodic flow from the impeller is steady with respect to the rotating system, and a jet flow zone and a slow flow zone exist side by side in a pitch of vanes. The streamlines on the rotating system are examined. If they flow out independently keeping the respective angular momentum, the backward leaning angle of the slow flow is larger than that of the jet flow as shown in Fig. 6, because the radial velocity component of the slow flow is smaller than that of the jet flow while their circumferential velocity components are nearly equal.

However, the slow flow and the jet flow must share a common border in the rotating system. Therefore, the jet flow pushes the slow flow perpendicular to the border, i.e. outward and forward. As a result the slow flow has both its circumferential and the radial velocity components increased. Losing its width it gradually unifies with the jet flow to finally form a uniform flow. In many cases unification is completed within a radius ratio of 1.2.

Unification of jet and slow flow was observed in many experiments and it was theoretically predicted, where unification is done by means of a pressure force in the rotating system. Therefore, it is an ideal work without loss of energy due to mixing.[11)] Apparently this is an excellent way to cure distorted velocity distribution around a rotating impeller. However, as the slow flow with increased circumferential velocity highly affected a larger friction force exerted by walls of the stationary diffuser, the pressure loss becomes larger unless the diffuser is very wide.

Cause of the pressure loss. Regarding the big pressure loss observed at the entry zone of the vane-less diffuser, there is a report[20)] that in many examples of distorted inlet velocity distributions, the pressure loss based on the above theory was not much different from the pressure loss evaluated by simply add the ordinary loss of mixing between the jet and the slow flow to the friction loss of the unified swirl flow on the wall. Probably the authors did not understand the mechanics of pressure force in a rotating system, and assumed that unification must accompany mixing loss.

There are unofficial reports in industry that the efficiency of an impeller was improved considerably by extending only the diameter of both the disc and the shroud ring of the rotating impeller. According to the theory[11)] it is quantitatively explained that in these cases, the big slow flow zone disappears between the extended disc and ring, avoiding the big friction force from the stationary diffuser walls.

Vane-less diffuser. Downstream of the diameter where flow is rotationally symmetric, it is desired to reduce the velocity as soon as possible. As swirl velocity varies inversely proportional to the radial location, a simple radial annular space between two parallel walls works as a diffuser. However, it requires a large outer diameter being accompanied with a large friction loss, which will limit the pressure recovery in the diffuser.

In spite of these demerits parallel wall diffusers have been widely used in industry, provided that they are the only diffusers suitable for a wide change of flow rate. Even so, if the flow rate is reduced too far, the radial

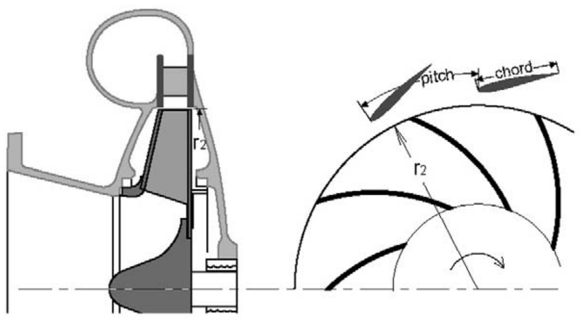

Fig. 7. Low solidity cascade diffuser.

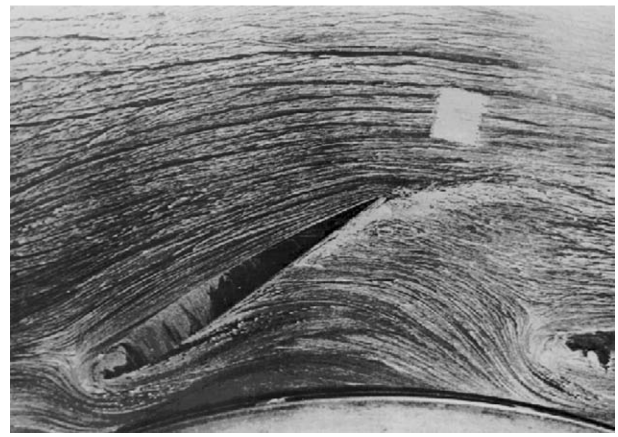

Fig. 8. Trace of path-lines on a side-wall of a cascade diffuser.

velocity component in the boundary layer on the wall is not sufficient to withstand the radial pressure gradient, and an inward cross-flow occurs near the wall.[21),22)] If the cross-flow exceeds a limit, the flow as a whole becomes unsteady[23)] and it is the lower limit of the flow rate for the compressor.

Low-solidity cascade diffusers. In order to achieve a high recovery rate of the dynamic pressure to the static pressure in a diffuser, it is desirable to install a set of guide vanes or a cascade of airfoils. However, a narrow throat between vanes must be avoided for operation at a wide range of flow rate as shown in Fig. 3. A low-solidity cascade diffuser (LSD) may satisfy these requirements, where low solidity means that the ratio of chord to pitch of vanes in a cascade is small. If the solidity is very small, LSD is not much different from a vaneless diffuser. On the other hand, the solidity should not be too large to make a narrow throat between adjacent vanes, which restricts the flow rate. Therefore, the value of solidity is very important.

Optimum solidity. Considering these conditions a cascade with solidity of about 0.7 was adopted in the diffuser as shown in Fig. 7 and it was tested hoping that the boundary layer on the both side-walls does not disturb the main flow.[24),25)] According to these experimental data, it worked effectively in a wide range of flow conditions.

At a small flow rate where the incidence angle of flow to the vanes increased up to 8 degrees, the lift coefficient on vanes in the LSD was about equal to the value on an isolated vane at the same inlet condition. As a result the dynamic pressure of flow through the LSD was satisfactorily recovered. At a large flow rate where the incidence angle of flow to the vane was negative, the influence of vanes on the flow was limited to only near the vanes. Consequently, ill effect due to the throat was hardly observed, because the area of the throat, the distance from the leading edge of a vane to the adjacent vane, was too large to restrict the flow.

Boundary layer on side-walls. In cases of vaneless diffusers, at a small flow rate the boundary layer on the sidewalls becomes very thick being accompanied with an rotationally uniform inward flow. In the present case with LSD, the trace of flow on the sidewall was visualized by means of oil film painted on the wall. The trace represents the flow near the wall in the boundary layer. According to the trace in Fig. 8 the path-lines on the sidewall, which start from the lower profile line of a vane, deviate from the main flow toward the adjacent vane, on the half way they change the direction inward and accumulate near the leading edge of the adjacent vane.

As the slow flow in the boundary layer accumulates and makes a hump, the pressure field drives it back toward the impeller and finally it mixes with the main flow from the impeller and it is completely energized. It can be said that the most of fluid in the boundary layer near the wall is continuously removed by the cross-flow and energized by the main flow. As a result, remainder of the boundary layer on the sidewall is thin and the main flow is not affected. Consequently performance of vanes is not disturbed by the boundary layer.

Regarding the flow downstream of the cascade of vanes, swirl of the main flow is weak by the force on vanes and the radial pressure gradient is moderate. Consequently the boundary layer on the sidewall remains thin. As a whole, LSD works fine in a wide range of flow rate.

Experimental results and applications. According to experimental data measured at a radial position of 1.3 times the radius of the impeller, at the design flow rate the recovery rate of the dynamic pressure delivered by the impeller was 33% in the case of a vane-less diffuser, while it was improved to 63% by a LSD with a solidity of 0.7, and even when the flow rate was doubled, the recovery rate was improved from 35% to 39%.[24),25)] In this connection, there is a report that in the case of a compressor, where the exit diameter of the vane-less diffuser was 1.6 times the diameter of impeller, by adopting LSD the exit pressure and the efficiency of the compressor were improved as much as 10% in the entire tested flow range, from 1 up to 1.5 times the design flow rate.[26)]

Furthermore, there is a report that a single stage transonic centrifugal compressor designed for a pressure ratio of 6.5 has demonstrated a stable wide operating range keeping high efficiency level.[27)] It has a velocity at the LSD inlet exceeding the sound velocity in spite of the common knowledge that the performance of vanes in a high speed flow is sensitive to the direction of flow relative to the vanes. As a whole, the low-solidity cascade diffuser is applicable to many centrifugal compressors including transonic compressors.[28),29)]

A company applied low-solidity cascade diffusers to more than a hundred units of centrifugal compressors including many 10~30 MW units over the last few years, and details of their excellent performance were reported in a book on turbomachines.[26)] Since then, low-solidity cascade diffusers have been well accepted by industry worldwide,[30)] and it is contributing to ecology by saving a tremendous amount of electricity. Also, there are reports on low-solidity cascade diffusers being applied in the fields of centrifugal pumps and turbo-superchargers.

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