

Review

Superalgebra and fermion-boson symmetry

By Hironari MIYAZAWA^{*1,†}

(Communicated by Toshimitsu YAMAZAKI, M.J.A.)

Abstract: Fermions and bosons are quite different kinds of particles, but it is possible to unify them in a supermultiplet, by introducing a new mathematical scheme called superalgebra. In this article we discuss the development of the concept of symmetry, starting from the rotational symmetry and finally arriving at this fermion-boson (FB) symmetry.

Keywords: supermultiplet, Lie algebra, hadron spectroscopy

Introduction

Symmetry is invariance under transformation. Right-left symmetry, the invariance under the space reflection, is one of the most elementary examples. Spherical symmetry is the invariance under the space rotation. Our three dimensional space is supposed to be invariant under the space rotation.

The above examples are transformations of space coordinates. There are other symmetries connected with transformations of the matter of our world. Proton and neutron are almost identical except for the charge. They can be regarded as the same particle, the nucleon, in two different states.

Lambda, one of the strange particles, differs in mass from the nucleon by 12 percent, and can hardly be regarded as the same. However, we can separate the hamiltonian into symmetric and asymmetric parts, first consider the symmetric part only and then consider the asymmetry. In this way the Lambda becomes a symmetry member of the nucleon. This symmetry is artificial and is not an intrinsic one given by god. Anyway, this way of separating the system into symmetric and asymmetric parts worked very well in strange particle physics.

So far, the space symmetry and the internal symmetry are separate and independent. Eugene Wigner combined these two into one symmetry and introduced the supermultiplet. In this scheme, nuclei

with different spins are grouped as symmetry members of a supermultiplet. Feza Gürsey and Luigi Radicati and Bunji Sakita extended the idea to include the strange particles. Here all important mesons are grouped in one supermultiplet, and baryons are in another.

The next question is: is it possible to incorporate mesons and baryons in one symmetry multiplet? Mesons obey Bose statistics and baryons obey Fermi statistics, so they cannot be regarded as the same particles. Nevertheless, by extending the notion of symmetry, even bosons and fermions can be grouped together. For this fermion-boson symmetry, the ordinary Lie group theory is insufficient. A new mathematical concept, called superalgebra, must be introduced. In the following sections we shall see how the idea of symmetry was extended to the FB symmetry.

Space symmetry

Our three dimensional space is spherically symmetric. The spherical symmetry means the invariance under the rotation, represented by the group SO(3). In this article we shall mainly use the term “invariance” to mean the invariance of the hamiltonian of the system. The rotation group SO(3) is mathematically equivalent to the special unitary group SU(2), where the fundamental representation consists of two complex elements.

$$\begin{pmatrix} q_{\uparrow} \\ q_{\downarrow} \end{pmatrix}. \quad [1]$$

Physically q_{\uparrow} and q_{\downarrow} are two states, the states of spin up and down of the fundamental spin 1/2 particle. Combination of these elements and its com-

^{*1} Professor emeritus, Faculty of Science, University of Tokyo, Tokyo, Japan.

[†] Correspondence should be addressed: H. Miyazawa, Faculty of Science, University of Tokyo, Hongo 7-3-1, Bunkyo-Ku, Tokyo 110-0003 (e-mail: miyazawa@phys.s.u-tokyo.ac.jp).

plex conjugate, or bound state of this particles and their antiparticles, produce group representations or multiplets of definite angular momentum. A multiplet is a set of degenerate states of a definite quantum number, the angular momentum this case. All members of a multiplet are the same particle, only the direction of its angular momentum being different in space. The term fundamental means that any spin state can be constructed from q_\uparrow and q_\downarrow and their antiparticles.

Space reflection is a separate transformation group. It is known that the space reflection symmetry only holds approximately, violated by the weak interaction. On the other hand, the rotational symmetry holds exactly, so far as there is no external field. In this article we shall mainly consider continuous transformations.

Symmetries in matter

Our space is spherically symmetric. Similar symmetry is found in the matter. The atomic nucleus consists of protons and neutrons. Proton and neutron are quite similar. Their masses differ only by 0.1 percent. Except for the electric charge and the small mass difference, proton and neutron are identical. They can be regarded as two different states of one particle, which is called nucleon. Similarly to eq. [1] we put

$$N = \begin{pmatrix} p \\ n \end{pmatrix}. \quad [2]$$

The dominant mass term in the hamiltonian can be written in the form

$$H_0 = M \sum_{\mathbf{k}} (a_{p\mathbf{k}}^\dagger a_{p\mathbf{k}} + a_{n\mathbf{k}}^\dagger a_{n\mathbf{k}}). \quad [3]$$

where a and a^\dagger are annihilation and creation operators, respectively, and \mathbf{k} is the momentum of the particle. This H_0 is invariant not only under the exchange of p and n but also under the SU(2) transformation of N (eq. [2]). It turned out, experimentally, that the whole hamiltonian is invariant under the SU(2). We introduce another three dimensional space, which is called the isospace. Proton and neutron form an isodoublet, a state of isospin 1/2, proton being the state of isospin up and neutron isospin down. The nuclear system is invariant by the rotation in the isospace. Here the small p-n mass difference and the coulomb interaction, which is small compared with the nuclear interaction, are neglected.

Charged and neutral pions form an $I = 1$ multiplet which is almost degenerate in mass. Experiments show that in reactions of nucleons and pions the isospin is conserved. An example is: The deuteron and the alpha particle are known to have isospin zero. Then the reaction

$$d + d \rightarrow \alpha + \pi^0, \quad [4]$$

can never happen if the isospin is conserved, the left-hand side being isospin 0 and the right-hand side being isospin 1. In fact, this reaction is found to occur very rarely.

Neglecting the electromagnetic interaction, which is not large compared with the strong interaction, the nucleon-pion physics is invariant under the rotation in the isospace. This is an experimental fact. So, we might say that this symmetry is given by god.

Strange particles

Lambda is a companion of the nucleon. It also has spin 1/2 and can turn into a nucleon. Further, lambda has a new quantum number, strangeness, which is conserved in the strong interaction. In addition to the fundamental elements proton and neutron (eq. [2]), we need a new element as the carrier of the strangeness.

$$B = \begin{pmatrix} p \\ n \\ \Lambda \end{pmatrix}. \quad [5]$$

This triplet cannot be fundamental. Other strange particles, sigma and xi are quite similar to nucleon and lambda and must be grouped together in one multiplet. Then the quark model is introduced. In this model the fundamental triplet consists of up quark u , down quark d , and strange quark s ,

$$Q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}. \quad [6]$$

Mesons are bound states of quark and anti-quark. Baryons, that is, nucleon, lambda and their brothers, are bound states of three quarks. We consider the symmetry among these particles.

Now that we have the fundamental triplet of elements, we want to consider the mixing among the three, the SU(3) group. u and d form an isodoublet and accept SU(2) transformation. However, s is much heavier than u and d , and the triplet does not form a degenerate multiplet.

Still, we wish to stick to the SU(3). We split the hamiltonian into SU(3) symmetric part and asymmetric part, and consider only the symmetric part, disregarding the asymmetric part as perturbation. This SU(3) symmetry is man-made, and is not given by god. We shall see how this artificial symmetry works in strange particle physics.

Group theoretically the nine states of $Q\bar{Q}$ split into an eight-dimensional representation and a one-dimensional one,

$$\mathbf{3} \times \bar{\mathbf{3}} = \mathbf{8} + \mathbf{1}. \quad [7]$$

Each representation corresponds to a degenerate multiplet. In fact, an octet of pseudoscalar mesons and a singlet of pseudoscalar meson have been found experimentally. For spin 1 bound states, also a singlet and an octet of vector mesons have been found.

Baryons are bound states of three quarks. Three quark products split into four representations.

$$\mathbf{3} \times \mathbf{3} \times \mathbf{3} = \mathbf{10} + \mathbf{8} + \mathbf{8} + \mathbf{1}. \quad [8]$$

The ten dimensional representation corresponds to the decuplet of spin 3/2 baryons. The eight corresponds to the spin 1/2 baryon octet. The rest, $\mathbf{8} + \mathbf{1}$ have not yet been established.

In this way all the known hadrons (baryons and mesons), can be classified as multiplets or representations of SU(3), but each multiplet is not degenerate. The mass difference among a multiplet member can be calculated by introducing the mass difference among the elementary triplet as perturbation. For the baryon octet, the mass formula is

$$\frac{1}{2}[M(N) + M(\Xi)] = \frac{1}{4}[3M(\Lambda) + M(\Sigma)]. \quad [9]$$

It is remarkable that the mass formula [9] is satisfied to within a percent of error. Similar formula can be derived for the octets of mesons, but they are not so well satisfied experimentally.

Thus, we see that the man-made SU(3) works well for the classification of hadrons and for deriving relations of their properties.

A charming idea is that intrinsically the hamiltonian is symmetric under the SU(3), but the symmetry is broken spontaneously. In this article, however, this possibility will not be considered.

Wigner's supermultiplet

Let us disregard the strange particles for a moment. We have two symmetries, the space rotation SU(2)_{Sp} and the isospace rotation SU(2)_{Iso}. These

two are independent. Wigner¹⁾ combined these two symmetries.

The fundamental elements are as follows:

$$\begin{pmatrix} p_{\uparrow} \\ p_{\downarrow} \\ n_{\uparrow} \\ n_{\downarrow} \end{pmatrix}. \quad [10]$$

Since the masses of the four elements are equal, the mass term in the hamiltonian is invariant under the SU(4) transformation. Thus Wigner extended the symmetry as follows:

$$\text{SU}(2)_{\text{Sp}} \otimes \text{SU}(2)_{\text{Iso}} \rightarrow \text{SU}(4)_{\text{SI}}, \quad [11]$$

The extended symmetry is applied to nuclear physics. Nuclei with different spins or isospins are classified in a supermultiplet. Wigner showed that this symmetry gave a good fit to light nuclear states where the Coulomb interaction is unimportant.

This idea of combining the space and internal symmetries was extended to the strange particle physics by Gürsey and Radicati²⁾ and by Sakita.³⁾ The fundamental elements are

$$q = \begin{pmatrix} u_{\uparrow} \\ u_{\downarrow} \\ d_{\uparrow} \\ d_{\downarrow} \\ s_{\uparrow} \\ s_{\downarrow} \end{pmatrix}, \quad [12]$$

and they consider the symmetry SU(6)_{SI}.

Mesons are bound state of q and \bar{q} . The thirty-six states decompose into two representations.

$$\mathbf{6} \times \bar{\mathbf{6}} = \mathbf{35} + \mathbf{1}. \quad [13]$$

The lowest-mass mesons, an octet of pseudoscalar mesons and a nonet of vector mesons, fit precisely to the **35**.

Baryons are bound states of three quarks.

$$\mathbf{6} \times \mathbf{6} \times \mathbf{6} = \mathbf{56} + \mathbf{70} + \mathbf{70} + \mathbf{20}. \quad [14]$$

The octet of spin 1/2 baryons and the decuplet of spin 3/2 baryon resonances fit precisely into the **56**.

In this way, all of the known mesons belong to one representation of the group SU(6), and the baryons belong to another representation. Members of each multiplet are not degenerate in mass. The symmetry is man-made and cannot be exact. A mass formula can be written for each multiplet, and these formulae are found to hold reasonably exactly. How-

ever, the most striking result is that the SU(6) theory gives the ratio of the magnetic moments of the proton and neutron to be

$$\frac{\mu_p}{\mu_n} = -\frac{3}{2}, \quad [15]$$

assuming that each quark has a magnetic moment proportional to its charge. The relation [15] holds almost exactly.

One additional comment is about the spin-statistics relation. A quark is a spin 1/2 particle and must obey Fermi statistics. The baryon's multiplet, **56** in eq. [13] is totally symmetric under permutations of the quarks, violating the statistics. This is evaded by introducing another internal variables called the color degrees of freedom. Each quark can have three color states, red, blue and green. The observed baryons are totally antisymmetric in the color variables so that they are totally symmetric in other variables.

Now that the color degrees of freedom have been introduced, we are tempted to consider a large group SU(18)_{SIC}, combining the color with other variables. However, the color degrees of freedom are never observed in nature.

Symmetry and Lie algebra

For continuous transformations, it is more convenient to consider infinitesimal transformations, or the generators of the group. The generators often have definite physical meaning: For instance, for SU(2)_{sp} the generators are the three components of the angular momentum.

If the hamiltonian H is invariant under a continuous transformation group, its generators G_i commute with H . If two quantities G_i and G_j commute with H , the commutator of G_i and G_j also commutes with H .

$$\begin{aligned} [G_i, H] &= 0, [G_j, H] = 0 \rightarrow \\ G_{\mathbf{k}} &= [G_i, G_j], [G_{\mathbf{k}}, H] = 0. \end{aligned} \quad [16]$$

So, the set of G_i 's forms a Lie algebra with the commutation relations of the form.

$$[G_i, G_j] = \sum c_{ij\mathbf{k}} G_{\mathbf{k}}. \quad [17]$$

This set generates the continuous symmetry group. A set of operators which commute with the hamiltonian defines a symmetry algebra.

Let us denote the creation and annihilation operators of the fundamental elements by $a_{\mathbf{k},i}^\dagger$ and

$a_{\mathbf{k},i}$, $i = 1, 2, \dots, n$. Here \mathbf{k} denotes the kinetic momentum of the particle. The mass term of the hamiltonian is

$$H_0 = M \sum_{i=1}^n \sum_{\mathbf{k}} a_{\mathbf{k},i}^\dagger a_{\mathbf{k},i}. \quad [18]$$

The operators

$$G_{ij} = \sum_{\mathbf{k}} a_{\mathbf{k},i}^\dagger a_{\mathbf{k},j}, \quad [19]$$

commute with H_0 and form a symmetry Lie algebra, which generates the unitary group U(n). The trace $\sum G_{ii}$ is the total number of particles, and the remaining G_{ij} form the special unitary algebra SU(n).

Fermion-boson symmetry

Starting from the rotational symmetry, SU(2)_{sp}, the concept of "symmetry" has been extended, partly artificially but supported by experiments. The resulting symmetry is SU(6). Here all important baryons are classified as a 56-dimensional multiplet, and the low lying mesons are classified as a 35-dimensional representation.

Can we go further? In other words, is it possible to classify baryons and mesons in one multiplet? Since baryons are fermions and mesons are bosons, it is not possible to mix them by ordinary transformations.

Let us assume that the fundamental particles include both fermions and bosons. Suppose that there are n fundamental Fermi-like elements (denoted by a_i, a_i^\dagger , $i = 1, \dots, n$) and m fundamental Bose-like elements (denoted by a_i, a_i^\dagger , $i = n+1, \dots, n+m$). Their masses are not the same, but we write the fundamental hamiltonian as follows:

$$H = M \sum_{\mathbf{k}} \sum_{i=1}^{n+m} a_{\mathbf{k},i}^\dagger a_{\mathbf{k},i} + \dots = H_0 + \dots \quad [20]$$

It may appear that H_0 is invariant under SU($n+m$) transformation, but this is not the case. Fermi-like element and bose-like element cannot be added; they cannot be mixed. However, "generators" can be defined as eq. [19], and they commute with H_0 .

$$G_{ij} = \sum_{\mathbf{k}} a_{\mathbf{k},i}^\dagger a_{\mathbf{k},j}, \quad G_{ij}^\dagger = G_{ji}, \quad [G_{ij}, H_0] = 0. \quad [21]$$

The operation of G_{ij} makes H_0 invariant and so the set of G_{ij} defines a symmetry of the system, although the set does not generate a continuous group.

If one of the (i, j) is Fermi-like and the other is Bose-like, that is, if $1 \leq i \leq n$, $n+1 \leq j \leq n+m$ or $1 \leq j \leq n$, $n+1 \leq i \leq n+m$, the G_{ij} is Fermi-like and commutators between them cannot be calculated but anti-commutators are,

$$\{G_{ij}, G_{kl}\} = \delta_{jk}G_{il} - \delta_{il}G_{kj}, \text{ Fermi-Fermi case.} \quad [22]$$

Otherwise, the commutators can be written as

$$[G_{ij}, G_{kl}] = \delta_{jk}G_{il} - \delta_{il}G_{kj}, \text{ other cases,} \quad [23]$$

or, in short,

$$[G_{ij}, G_{kl}]_{\pm} = \delta_{jk}G_{il} - \delta_{il}G_{kj}. \quad [24]$$

Relation [24] contains both commutators and anticommutators so the set of operators G_{ij} does not form a Lie algebra. An algebra with anticommutation as multiplication is called a Jordan algebra. Our system is a mixture. Multiplication is defined by the anticommutator between Fermi-like elements and by the commutator for other combinations.

Our “superalgebra”⁴⁾ does not generate a continuous group. Still, from the commutation-anticommutation relations [24], we can construct representations, determine multiplets and calculate matrix elements. The operators G_{ij} and the relations [24] are similar but, of course, not equal to the generators of the special unitary group of $n+m$ dimensions. We call this superalgebra $SU(n|m)$.

In this way fermion-boson symmetry can be achieved by introducing a superalgebra. Fermions and bosons can be grouped in a multiplet. Each fermion and boson in a supermultiplet are the same particle in different phases.

SU(6|21) Symmetry

Having arrived at the idea of supersalgebra, we try to extend the S(6) symmetry to include baryons and mesons in one supermultiplet. In addition to the elements of SU(6), eq. [12], a set of Bose particles must be included in the fundamental elements. This set of bosons must be an SU(6) multiplet by themselves. The simplest possibility is a singlet spin 0 particle but this is insufficient. The next possibility is **15** or **21**, and we see that the 21-dimensional multiplet made from $\bar{q}q$ system is most convenient.⁵⁾ The **21** representation consists of an SU(3) triplet of scalar mesons and a sextet of axial-vector mesons. These, together with the triplet of quarks (eq. [12]), form the fundamental elements of SU(6|21).

$$F = \left(\frac{\mathbf{6}}{\mathbf{21}} \right). \quad [25]$$

The adjoint representation of SU(6|21) or $F\bar{F}$ consists of, in terms of the SU(6) multiplets,

$$\begin{aligned} (\mathbf{27}, \mathbf{27}) &= (\mathbf{6} + \mathbf{21}, \mathbf{6} + \mathbf{21}) \\ &= \mathbf{1} + \mathbf{35} + \mathbf{56} + \mathbf{56} + \mathbf{70} + \mathbf{70} + \mathbf{1} \\ &\quad + \mathbf{35} + \mathbf{405}. \end{aligned} \quad [26]$$

The first two terms, **1** and **35**, are the well-established negative-parity mesons, that is, pseudoscalar singlet, pseudoscalar octet, vector singlet and vector octet mesons.

The next **56** is the baryon octet and decuplet. The **56** is their antiparticles. **70** and **70** are multiplets of baryons and antibaryons, but they are not yet established. The remaining **1** + **35** + **405** are positive parity mesons. **1** + **35** contains singlet and octet of scalar mesons and singlet and octet of axial-vector mesons. **405** contains tensor mesons which are not yet established. In this way, all of the well-established hadrons are classified in a supermultiplet of SU(6|21).

The Bose elements **21** added to the triplet of quarks to form SU(6|21) are actually two antiquark states, and cannot be regarded as fundamental. This symmetry is not intrinsic but is artificial. However, by using this FB symmetry, all known hadrons (baryons and mesons) are classified as the same, and relations between scattering amplitudes, for instance, can be derived. The symmetry is broken, of course. Diquarks will be heavier than quarks. Then we have a relation among the masses of negative parity mesons, positive parity mesons and baryons:

$$M(M_-) + M(M_+) = 2M(B). \quad [27]$$

This relationship seems to be roughly satisfied experimentally.

One comment is that in this SU(6|21) scheme, there is no problem of statistics, and the color degrees of freedom are not necessary. All elements can have integral charges.

Is our space fermion-boson symmetric?

Fermion-boson symmetry claims that a supermultiplet contains both fermions and bosons. This is a convenient and useful way to classify and handle the existing hadrons. This is an artificial symmetry and cannot be regarded as intrinsic one given by god.

Our three dimensional space is spherically sym-

metric. Is it possible that the space is fermion-boson symmetric, either intrinsically or artificially?

The simplest superspace can be created from the fundamental elements,

$$P = \begin{pmatrix} q_{\uparrow} \\ q_{\downarrow} \\ s_0 \end{pmatrix}, \quad [28]$$

where the s_0 stands for a spin 0 particle. The symmetry algebra $SU(2|1)$, from eq. [28], is in parallel to the $SU(3)$, eq. [6], which generates the internal symmetry. If our space is FB symmetric as is given by $SU(2|1)$, every particle must have its super partner. For instance, electron must have boson partner of similar mass. No such boson has been found.

It is proposed that the space FB symmetry holds at very short distance, that is, at very large energy scale. All known particles are members of a supermultiplet of zero-mass states, although they are non-degenerate due to small symmetry breaking. Nothing is wrong with this scheme. However, relations similar to eqs. [9], [15] or [27] are needed to support this proposal.

Theoretically, the introduction of a spin 0 particle as a fundamental element is not without doubt. In the case of internal symmetry, the third element, Λ in eq. [5] or s in eq. [6], was necessary as a carrier of the conserving quantum number strangeness. With regards to the symmetry of the space, there is no such necessity. All spin states can be created from spin 1/2 particles and its antiparticles and the spin 0 object is not necessary. The number of the fundamental elements must be as small as possible.

We have been considering the space rotation $SU(2)_{\text{sp}}$. The Lorentz group $SL(2, C)$ can be made Fermion-boson symmetric by introducing boson element in addition to the fundamental two-component spinor. Here the symmetry means the invariance of the lagrangean of the system and not of the hamiltonian. The symmetry can be defined by a superalgebra, that is, by commutation and anticommutation relations. This scheme is called supersymmetry.⁶⁾

It is to be noted that Wigner's symmetry (eq. [11]), and the subsequent extensions ($SU(6)$ and $SU(6|21)$), cannot be made relativistic. The internal variables transform as a representation of the special unitary group while Lorentz spinor transforms as a representation of the special linear group, and they cannot be mixed in a larger group. The fermion-boson symmetry can be relativistic, in so far as the

added elements are lorentz covariant quantities and independent of internal symmetry. The total symmetry is the direct product of the internal symmetry and the relativistic space symmetry.

Conclusion

Fermion-boson symmetry attempts to regard fermions and bosons as the same particles. Ordinary symmetry is defined as invariance under a transformation group. In the case of the fermion-boson symmetry, there is no transformation group. The symmetry is defined not by a group but by a superalgebra, where the multiplication is defined by anticommutator in some cases and by commutator in others. A representation of the superalgebra (that is, a supermultiplet) contains both fermions and bosons. For example, all important hadrons can be classified in one supermultiplet.

While the nonrelativistic fermion-boson symmetry works well in hadron spectroscopy, the relativistic FB symmetry fails to do so. If the space is fermion-boson symmetric, all particles must have their super partner of different statistics. No such partners are found. It is possible to expect that the space FB symmetry is seriously broken but would hold at very short distances. We can split the system into FB symmetric part and symmetry breaking part, and first consider the symmetric part. To see if this man-made symmetry works, further investigation is necessary.

References

- 1) Wigner, E. P. (1937) On the consequences of the symmetry of the nuclear hamiltonian on the spectroscopy of Nuclei. *Phys. Rev.* **51**, 106–119 (1937).
- 2) Gürsey, F. and Radicati, L. A. (1964) Spin and unitary spin independence of strong interactions. *Phys. Rev. Lett.* **13**, 173–175.
- 3) Sakita, B. (1964) Electromagnetic properties of baryons in the supermultiplet scheme of elementary particles. *Phys. Rev. Lett.* **13**, 643–646.
- 4) Miyazawa, H. (1966) Baryon number changing currents. *Prog. Theor. Phys.* **36**, 1266–1276.
- 5) Miyazawa, H. (1968) Spinor currents and symmetries of baryons and mesons. *Phys. Rev.* **170**, 1586–1590.
- 6) Wess, J. and Zumino, B. (1974) Supergauge transformation in four dimensions. *Nucl. Phys.* **B70**, 39–50.

(Received Dec. 18, 2009; accepted Jan. 20, 2010)

Profile

Hironari Miyazawa was born in 1927, in Tokyo. He graduated from University of Tokyo in 1950 and obtained Dr. of Science Degree in 1953 under the supervision of Prof. Takahiko Yamanouchi. From 1953 to 1955 he was a research associate at Institute for Nuclear Studies, University of Chicago, where he made research on theoretical nuclear physics under Prof. Gregor Wentzel and Prof. Enrico Fermi. After returning to University of Tokyo he was promoted to Professor of Physics in 1968. In 1988 he moved to Kanagawa University and served there until 1998. During these periods he also served visiting professorship at University of Chicago and University of Minnesota, and directorship at Meson Science Laboratory, University of Tokyo. Currently he is a Professor Emeritus, University of Tokyo.

